

Notes added by Frank Robbins.

The two equations for $b_{0.1}$ may be derived from equations (4) and (7) respectively as given by Verhulst (*loc. cit.*) if it is remembered that his D is the complete elliptic integral of the first class

$$\int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{1 - c^2 \sin^2 \phi}}$$

and his b is the complementary modulus $\sqrt{1 - c^2}$.

The second and more accurate formula may be slightly simplified—it easily becomes

$$b_{0.1} = \frac{3^2}{\{1 + \alpha_1^{\frac{1}{2}} + \sqrt[4]{8(\alpha_1^{\frac{1}{2}} + \alpha_1^{\frac{3}{2}})}\}^2} \{1 + 2q^{16} + 2q^{64} + \dots\}^2;$$

but even so, it is somewhat troublesome, and the first formula would probably be adopted by most computers. It is certainly quicker even if a special value of N_1 is computed instead of being interpolated from the table, page 649.

COMPUTATION OF N_1 .

The exact value of N_1 is $\log_{10} (1 + 2q^4 + 2q^{16} + 2q^{36} + \dots)^2$
or $\log_{10} (1 + 4q^4 + 4q^8 + \dots)$.

If α is not greater than 0.85 it will be sufficient to stop at $4q^4$, and then it is easy to see from the equations

$$\alpha^2 + \alpha_1^2 = 1 \quad \frac{l}{2} = \frac{1 - \alpha_1^{\frac{1}{2}}}{2(1 + \alpha_1^{\frac{1}{2}})}$$

$$q = \frac{l}{2} + 2\left(\frac{l}{2}\right)^5 + 15\left(\frac{l}{2}\right)^9 + \dots$$

that $N_1 = m \log_e (1 + 4q^4 + 4q^8 + \dots)$

or $N_1 = 4mq^4$ very nearly

$$N_1 = 4m \left\{ \left(\frac{l}{2}\right)^4 + 8\left(\frac{l}{2}\right)^8 \right\} \quad m \text{ being } 0.4342945.$$

This is a very convenient working formula, and it can be computed for any given value of α in less than ten minutes. Zech's addition table serves admirably here; the argument being the *difference of the logarithms* of the two terms within the brackets. This *difference* can be found mentally from the log of the first term $\left(\frac{l}{2}\right)^4$ without the $\log 8\left(\frac{l}{2}\right)^8$ being known. It is merely necessary to add 0.9030900 to the mantissa of $\log \left(\frac{l}{2}\right)^4$ and to

subtract the sum from the characteristic with changed sign. This is perhaps quite a minor point, but still it will be of interest to computers, inasmuch as it saves half the figures (say 20 figures if using 7 fig. logs), and with every figure saved a possible error is avoided.

For the Gaussian relations of Section II. the reference is to *Disquisitiones circa seriem infinitam*: Werke, vol. iii. p. 130, Göttingen, 1876.

N_1 expressed in Units of the Eighth Decimal Place.

α	0	1	2	3	4	5	6	7	8	9
0.4	3	3	4	5	6	7	9	10	13	15
0.5	18	22	26	31	37	45	53	63	75	89
0.60	105	107	108	110	112	114	116	118	120	122
.61	124	126	128	130	132	135	137	139	141	144
.62	146	149	151	154	156	159	161	164	167	170
.63	172	175	178	181	184	187	190	193	197	200
.64	203	207	210	214	217	221	224	228	232	236
.65	239	243	247	251	256	260	264	268	273	277
.66	282	287	291	296	301	306	311	316	321	326
.67	332	337	343	348	354	360	366	372	378	384
.68	390	397	403	410	416	423	430	437	444	452
.69	459	466	474	482	490	498	506	514	523	531
.70	540	549	558	567	576	586	595	605	615	625
.71	635	645	656	667	678	689	700	712	723	735
.72	747	759	772	785	798	811	824	837	851	865
.73	879	894	909	924	939	954	970	986	1002	1019
.74	1036	1053	1070	1088	1106	1124	1143	1162	1181	1201
.75	1221	1241	1262	1283	1304	1326	1348	1370	1393	1416
.76	1440	1464	1488	1513	1539	1565	1591	1618	1645	1672
.77	1700	1729	1758	1788	1818	1849	1880	1912	1944	1977
.78	2011	2045	2080	2115	2151	2188	2225	2264	2302	2342
.79	2382	2423	2464	2507	2550	2594	2639	2684	2731	2778
.80	2826	2875	2926	2976	3028	3081	3135	3190	3246	3303
.81	3361	3420	3481	3542	3605	3669	3734	3800	3868	3937
.82	4007	4079	4152	4226	4302	4380	4459	4539	4622	4705
.83	4791	4878	4967	5058	5151	5245	5342	5440	5540	5643
0.84	5747	5854	5963	6074	6188	6304	6422	6543	6667	6793

	$\log p_1$	$\log p_2$	$\log p_3$	$\log p_4$	$\log p_5$	$\log p_6$	$\log p_7$	$\log p_8$
α	9'698 9700	9'875 0613	9'920 8187	9'942 0081	9'954 2425	9'962 2114	9'967 8153	9'971 9713
	+	+	+	+	+	+	+	+
0'00	0	0	0	0	0	0	0	0
'02	217	72	36	22	14	10	8	6
'04	869	290	145	87	58	42	31	24
'06	1958	653	327	196	131	94	71	55
'08	3484	1162	582	349	233	167	125	98
'10	5452	1819	910	546	364	260	195	152
'12	7867	2627	1315	790	527	377	283	220
'14	10733	3585	1796	1079	720	515	386	301
'16	14056	4698	2354	1415	944	675	507	394
'18	17843	5968	2992	1799	1201	859	645	502
'20	22105	7400	3712	2233	1491	1067	801	624
'22	26848	8996	4516	2717	1816	1299	976	760
'24	32086	10761	5407	3255	2176	1557	1170	911
'26	37829	12701	6386	3847	2572	1842	1384	1078
'28	44093	14822	7460	4497	3009	2155	1619	1262
'30	50893	17130	8630	5207	3485	2498	1878	1463
'32	58245	19632	9902	5979	4004	2871	2159	1683
'34	66170	22336	11279	6815	4568	3276	2465	1922
'36	74687	25253	12767	7724	5179	3717	2798	2182
'38	83821	28388	14374	8703	5842	4195	3159	2465
'40	93598	31757	16102	9760	6556	4710	3549	2770
'42	104048	35372	17963	10900	7328	5269	3972	3103
'44	115200	39245	19963	12128	8161	5872	4429	3461
'46	127094	43392	22111	13451	9060	6523	4923	3849
'48	139769	47832	24420	14876	10031	7229	5460	4271
'50	153269	52583	26899	16410	11078	7990	6039	4726
'52	167645	57666	29562	18064	12208	8813	6666	5220
'54	182955	63109	32426	19847	13432	9707	7348	5759
'56	199262	68939	35507	21772	14754	10674	8088	6343
'58	216637	75186	38823	23850	16186	11723	8891	6978
'60	235166	81889	42400	26102	17743	12867	9768	7673
'62	254942	89090	46262	28542	19434	14112	10725	8433
'64	276075	96838	50442	31194	21279	15474	11774	9266
'66	298692	105191	54974	34082	23294	16966	12926	10184
'68	322939	114214	59900	37236	25503	18606	14195	11196
'70	348993	123989	65273	40694	27935	20418	15601	12321
'72	377063	134611	71154	44500	30623	22427	17163	13574
'74	407397	146196	77615	48705	33606	24666	18911	14979
'76	440299	158883	84749	53377	36938	27176	20877	16564
'78	476145	172847	92669	58598	40680	30007	23102	18363
'80	515418	188318	101526	64481	44922	33232	25648	20428
'82	558714	205572	111502	71156	49766	36933	28580	22816
'84	606855	225003	122861	78827	55370	41241	32011	25622
'86	660920	247122	135942	87740	61932	46316	36074	28958

June 1909.

Errata.

651

*Errata in Mr. Hinks' Tables for computing Standard Co-ordinates,
Memoirs of R.A.S., vol. lvii. p. 153.*

Arg. 248 for 6243 read 6232.

„ 249 „ 6222 „ 6202.

„ 255 „ 6014 „ 6017.